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Summary

To avoid high profit levels often experienced in countries where monopolies in public utility sectors are regulated through price-cap mechanisms, several regulatory agencies have recently introduced profit-sharing (PS) clauses aimed at obtaining price reductions to the benefit of consumers. However, the implementation of these PS clauses has often turned out to be severely con- trained by the incompleteness of the price-cap itself and the non-verifiability of firms'profits. This paper studies the properties of a second-best optimal PS mechanism designed by the regulator to induce the regulated monopolist to divert part of its profits to custormers. In a dynamic model where a reg- ulated monopolist manages a long-term franchise contract and the regulator has the option to revoke the contract if there are serious welfare losses, we first derive the welfare maximising PS mechanism under verifiability of prof- its. Subsequently, we explore the sustainability of the PS mechanism under non-verifiability of profits. In a infinite-horizon game, it is showed that the dynamic sustainability of the PS clause crucially depends upon the magni- tude of the regulator's revocation cost: the higher this cost, the lower the profit shared and the less frequent the regulator's PS introduction. Finally, we present the endogenous and dynamic price adjustment which follows the adoption of the investigated PS mechanism in a price-cap regulation setting.

Keywords: Price-cap regulation, Profit-sharing, Real options

JEL Classification: C73, L33, L5

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1 Introduction

Recent European liberalization and US experience in regulation of public utilities shows that price-cap regulation (PCR) allows prices to diverge greatly from actual costs and often generate "abnormal" profits for the firm.[1] Regulators dislike high corporate profits under PCR because they reduce the welfare of consumers and - favouring the firm - downgrade the regulator's reputation for being able to set the "right" price of the service. This is why, in the last decade, PCRs have been modified in a variety of ways in order to induce the regulated firms to rebate part of their profits to customers. Precisely, regulators have often advocated "profit-sharing" (PS) - or "earning-sharing" - schemes to make the regulated firm share with its customers a fraction of the profits it generates beyond a certain level.

In the European experience of regulation, the textbook example for profit-sharing refers to the price-cuts implemented by the British electricity regulator in between 1994 and 1995, well before the official price review in 1999: since the initial price control for the electricity companies turned out to be over-generous allowing high profits, the regulator intervened reducing prices and thus returning some of those "excess" profits to consumers.[2] Sappington (2002), among others, shows that these PS practices are usually introduced in the US telecommunication industry by the regulator in the form of direct payment to customers or reduced prices for key services.[3] The present paper mainly deals with this latter form of PS.

A fundamental feature of these real-world PS mechanisms is, thus, the discretion left in the hands of the regulator which entitles the regulator himself to adjust the PCR adopted ex-ante calling for an unilateral "renewal" of the regulatory contract. By its own discretionary nature, though, the PS mechanism allows for disputes and ex-post re-contracting between the regu-

[1] This drawback of the price-cap as an incentive mechanism stems from its inability to set a contingent price that incorporates all the uncertainties faced by the firm in each period of the regulatory contract.

[2] Similar price-cuts have since been made by other regulators as British Gas Transco (gas transmission and distribution), National Grid Company (electricity transmission) (Green, 1997). Other often quoted European examples are about Oftel, the British tlc regulator (Armstrong et al.(1994)), and Ofwat, the water industry regulator in England and Wales (see the Ofwat's home page for the current profit-sharing mechanism).

[3] Reduction in prices has been widely employed to regulate intrastate accounting rates affecting earnings of telecommunication providers (Sappington, 2002).

lator and the regulated firm about both the level of profit that should trigger the sharing rule and the dynamic path that the regulated price should follow. It has been informally argued that this may make it substantially more difficult to implement PS prescriptions.[4]

The aim of this paper is to investigate the properties of second-best optimal PS mechanisms, designed by regulators to induce regulated firms to divert part of their "excessive" profits to custormers. Specifically, we analize a dynamic game with two players: a regulated monopolist who manages a long-term franchise contract to provide a public utility service (e.g.: water supply, waste managment, gas or electricity distribution, etc.); and a welfare-maximising regulator who has the right, during the contractual relationship, to ask for price reductions from the regulated firm, and to revoke the franchise contract if he perceives the firm's profit as "excessively" high. We model the regulator's "outside" option to revoke the contract as a perpetual call option where the regulator - considering the firm's profit as an underlying asset - has to determine when to pay an exercise price to re-acquire managment of the utility and to re-determine provision of the service. Specifically, the game may last an infinite number of periods, and ends once the regulator exercises the option to revoke the franchise contract. Each period is divided into four stages: at the first stage, nature chooses the realization of a random variable, determining the firm's profit; at the second stage, after having observed the firm's profit, the regulator decides whether or not to ask for a price reduction; at the third stage, the firm decides whether or not to comply with the regulator's prescription; finally, at the fourth stage, the regulator, based on the price set by the monopolist, may revoke the contract.

It turns out that the assumption on profit verifiability affects the analysis of the game beginning with the second stage: indeed, when the profit - and the other regulatory variables - are observable but nonverifiable, the regulator cannot force the firm to cut "excessive" profits as "no court or other third party will accept to arbitrate a claim based on the value taken by these variables"[5]. This implies that - when profit are not verifiable - the firm can effectively choose whether to comply or not with the PS rule, retaining all profits above the profit threshold that triggers the PS until the regulator revokes the contract. In contrast, when the verifiability of profit is assumed,

[4] As stressed by Green (1997), the PS would require audited cost information for calculating allowable profits (and prices) levels, information which are often very difficult to be collected by the regulator.

[5] Salanié (1997, p. 177).

the regulator's PS prescription reduces the game to a take-it-or-leave-it offer to the firm.

For the sake of clarity, the analysis is broken into two parts. As a benchmark, we firstly investigate the simpler, though less realistic, case where the firm's profit is verifiable and - consequently - the PS mechanism imposes contractual obligations contingent on realized profits; then, we move to the more realistic case of profit's non-verifiability.

In the first part of the analysis, after having formally defined the PS clause we investigate, we identify the profit threshold that determines the regulator's introduction of the welfare-maximizing PS rule. We then demonstrate that at such a profit level the regulator is indifferent between contract closure and imposing the PS. Hence, we formally show that in the unique equilibrium of the game the monopolist will always comply with the regulator's PS rule and the contract will never be revoked; the regulator will impose the PS rule whenever the profit is higher than the identified trigger level.

In the second part of our analysis we turn to the case of profit non verifiability. The main difference with the previous case is that a monopolist, who decides not to comply with the PS rule, can - now - retain all the profits above profit's threshold triggering PS until the regulator calls for contract closure. The regulator will now revoke the contract, say at period t, only if revocation at that period is effectively his best reply. In other words, in the absence of profit verifiability, an incentive constraint imposing dynamic optimality of the revocation policy must be satisfied in order to make credible the regulator's revocation threat.

In this setting, we formally show that for all the profit levels higher that the profit threshold which makes the regulator indifferent between contract closure and imposing the PS, it is sequentially optimal for the regulator to revoke the contract, while revoking the contract for lower profit levels will never satisfy the sequentiality. Hence, the perfect equilibrium of the game with profits non-verifiability is also such that the firm complies with the PS rule chosen by the regulator in each period, as long as the revocation has not been carried out.[6]

In equilibrium, the expectation of being able to induce profit-sharing makes it rational for the regulator not to exercise its option to revoke, and this fact also makes it rational for the firm to continue to comply with the

[6]Efficient sub-game perfect equilibria in infinite-horizon threat-games are investigated in Klein and O'Flaherty, 1993; Shavell and Spier, 2002.

PS prescription. On the one hand, given that for the monopolist the loss from revocation of the (infinitely-lived) contract is greater than the expected stream of profit cuts (prescribed by the PS rule), it will be efficient for the firm to continuosly maintain profit at a level lower than the threshold that triggers the PS. On the other hand, the regulator will revoke the contract for any profit level higher than the profit threshold that triggers the PS.

Our model also shows that the sustainability of the PS mechanism crucially depends upon the magnitude of the regulator's revocation cost. Such cost represents a form of capture of the regulator by the firm[7]: the higher the revocation cost, the lower the profit shared and the less frequent the regulator's PS prescription.

Finally, as in our model the firm's profit is stochastically determined, the profit threshold that triggers the PS rule defines an (expected) time interval after which the monopolist will be asked for the first time to reduce its price and a time interval between each pair of regulatory reviews asking for price reductions. We show that both the equilibrium trigger profit level as well as the time interval between each pair of regulatory reviews (i.e. the regulatory lag) positively depend on the regulator's revocation cost.

The present paper is related to two different strands of literature. On a formal level, our paper builds upon the literature on the stochastic control techniques developed to identify optimal timing rules and optimal barrier regulations.[8] These techniques are widely used in the literature of irreversible investments,[9] and emphasize the option value of delaying investment decisions, i.e. the value of waiting for better (although never complete) information on the stochastic evolution of a basic asset.

In reference to the economic literature on the regulation of firms, our paper takes stock of the studies on drastic regulatory changes such as stochastic regulatory review (Bawa and Sibley, 1984) and expropriation by the regulator (Salant and Woroch,1992; Gilbert and Newbery, 1994). Bawa and Sibley (1984) show that the firm's incentive to indulge in over-capitalization can be tempered by the fact that this raises profits and - consequently - makes it more likely that the regulator will cut prices; in contrast to their approach, which emphasizes strategic firm behaviour *vis a vis* both the likelihood regulator review and price adjustment, we focus on the regulator's decision to

[7]For a discussion on the large literature on regulatory capture we refer to Laffont and Tirole (1994, chapt. 11).

[8]See Harrison and Taksar (1983), Harrison (1985) and Dixit (1993).

[9]Dixit and Pindyck (1994) is the seminal text in this area.

impose a regulatory review in the form of a PS rule and on the informational conditions which makes it enforceable.

Salant and Woroch (1992) and Gilbert and Newbery (1994) present models on expropriation by the regulator where the price regulation occurs endogenously as a self-enforcing and mutually beneficial cooperative equilibrium. In both those discrete-time repeated game frameworks, the regulatory lag does not affect the players' behaviour. In contrast, in our continuous-time repeated game model, the explicit unilateral approach to contract renewal[10] - i.e. the regulator sets the PS rule or calls for contract closure - allows us either to determine the regulatory lags endogenously or to study its determinants. Specifically, in our framework, the endogeneity of the regulatory lag[11] consistently belongs both to the level of the firm's profit and the regulator's revocation cost.

According to the approach of this paper, the regulator's decision to introduce the PS rule is based on welfare consideration and modelled along with the option to revoke the contract: both these elements are absent - as far as we know - from the literature on PS and expropriation by the regulator. Indeed, these considerations allow us to recognize, firstly, that the PS calls for an intertemporal regulatory setting and, secondly, that market uncertainty and the regulator's credible threat to close the contract, represent relevant issues in the PS definition and its enforcement.

Finally, we ought to mention a limit of the present analysis: we do not consider in this paper the well-known trade-off generated by the introduction of a PS rule between lowering extreme profits and dulling the firm's incentive for cost reduction.[12] This is because our model, though simple, focusses on the essential features which characterize the regulator's motivation and the information setting for enforcing a PS rule, leaving the firm the strategic choice of compling with the PS rule or not complying.

The remainder of the paper is structured as follows. Section 2 presents

[10] For an analysis of incentives to call for contract renewal from a bilateral perspective, see Andersen and Christensen (2002).

[11] As discussed in Laffont and Tirole (1994, p.15), endogeneity of the regulatory lag - which is in the essence of the most real-life adopted regulation mechanism - is especially important when the incentive properties of regulation are investigated.

[12] In this respect the literature has mainly stressed that compulsory sharing of profit may: a) reduce the firm's incentive to minimize operating costs and increase revenue (Lyon, 1996; Crew and Kleindorfer, 1996); b) provide an incentive to undertake projects that are unduly risky (Blackmon, 1994); c) lead the utility to delay investment (Moretto et al., 2003).

the basic model in which a PS rule is introduced. Section 3, derives the regulator's value of the option to revoke the contract as well as the optimal profit threshold that triggers it (*Proposition 1*): when firm's profits are verifiable, this threshold indeed represents the optimal level at which to introduce the PS rule (*Proposition 2*). Section 4 explores PS sustainability when the firm's profit is nonverifiable (*Proposition 3*). Section 5 investigates the price adjustment which follows the introduction of a PS rule in a PCR setting, and finally Section 6 concludes with some implications of the study for policy and a suggestion of how the model could be extended.

2 The model

We consider a risk-neutral profit-maximizing firm managing a one-time sunk indivisible project for the provision of a public utility under a long-term franchise contract. We assume that no new investments[13] are undertaken during the contract period; this assumption emphasizes that the focus of our analysis is on the firm-regulator relationship in a regulatory framework designed to cope with high profits, not with conduct that aims to conceal high profits.

Moreover, we assume that the infinitely lived project produces a flow of profits π_t which evolves over time according to a geometric Brownian motion, with instantaneous rate of growth $\alpha > 0$ and instananeous volatility $\sigma \geq 0$:

$$d\pi_t = \alpha\pi_t dt + \sigma\pi_t dW_t, \quad \pi_0 = \pi \tag{1}$$

where dW_t is the standard increment of a Wiener process, uncorrelated over time and satisfying the conditions that $E(dW_t) = 0$ and $E(dW_t^2) = dt$. Hence, under these assumptions the value of an infinite project, $V(\pi)$, can be expressed by (Harrison, 1985, pag.44):

$$V(\pi) = E_0\left\{\int_0^\infty \pi_t e^{-\rho t} dt \mid \pi_0 = \pi\right\} = \frac{\pi}{\rho - \alpha} \tag{2}$$

where $\rho > \alpha$ is the constant risk-free rate of interest[14]. As V is a multiple of π, it also is a geometric Brownian motion with the same parameters α and

[13] Moretto et al. (2003) investigate endogenous investment in profit-sharing regulation.

[14] Alternatively, we could use a discount rate that includes an appropriate adjustment for risk and take the expectation with respect to a distribution for π that is adjusted for risk neutrality (see Cox and Ross, 1976; Harrison and Kreps, 1979).

σ:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t, \qquad \text{with } V_0 = V \tag{3}$$

Although equation (2) is an abstraction from real projects, we can think at π as the "reduced form" of a more complex model where the instantaneous cash flow $\pi_t = \pi(\mathbf{z}_t)$ depends on a vector of variables $\mathbf{z}_t$, which may include the market price, the quality of the service, the firm's investments and market shocks that account for some sources of uncertainty in consumer demand and/or technological choice.

In our model, when the monopolist makes "huge" profits, the regulator can introduce a PS rule to divert these "excess" profits to consumers, or revoke the firm's contract to re-obtain responsability for managing the utility and re-address the project's profitability. In what follows, we firstly model how the PS rule is designed and, in the next section, how the contract closure is modelled.

Among the many ways of introducing PS, the simplest one is the setting of an upper bound on profits by the regulator, π^*.[15] In technical terms π^* is a reflecting barrier, i.e. at π^*, a "profit cut" stops π_t from going above π^*.

To set up an appropriate mathematical model representing the above PS rule we are guided by the theory of optimal barrier regulations (Harrison and Taksar, 1983; Harrison, 1985). In addition, as from (2) choosing π^* is equivalent to choosing an upper limit to the value of the project V^*, hereinafter we take V_t as the primitive exogenous state variable for the regulatory process we are considering. Thus, if the monopolist starts with the initial project's value $V_0 < V^*$, the regulator's PS rule applies as follows:[16]

- for $V_t < V^*$, let V_t evolve on its own and follow the geometric Brownian motion (3);

- for $V_t \geq V^*$, introduce the PS rule dr_t to stop V_t from going above V^*. The new "regulated" process $V_t - r_t$ can be described by the following

[15] For qualitatively analogous rules see Sappington and Weisman (1996).For a more general discussion about profit-sharing rule adopted in the recent experience of public utility regulation see Sappington (2002)

[16] Really, this is a "value-sharing" rule: we call it PS as there is a one-to-one relationship between the firm's value and profits. See Moretto and Valbonesi (2000) for the explicit model of the production decision of the firm.

stochastic differential equation[17]:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t - dr_t, \quad V_0 = V, \text{ for } V_t \in (0, V^*] \qquad (4)$$

where the increment dr_t represents the firm's profit reduction between time t and $t + dt$.

The PS rule is then a process proportional to V_t, parametrized by the initial condition V^*, right-continuous, non-decreasing and non-negative, defined as:

$$r_t = \ a(V^*)V_t \qquad \text{if } V_t \geq V^*, \qquad (5)$$

where $a(V^*) \equiv [1 - \inf_{T^* \leq v \leq t}\left(\frac{V^*}{V_v}\right)]$, $T^* = \inf(t \geq T^* \mid V_t - V^* = 0^+)$ and $r_t = 0$ for all $t \leq T^*$(see Appendix A). As shown in Figure 1 below, the PS defined in (5) increases to keep V_t lower than V^* and is given by the cumulative amount of profit control exerted on the sample path of V_t up to t.[18]

Figure 1 - about here -

[17] Stochastic differential equations such as (5) are a notational convenience, since only their integral counterparts are well defined. The "impulse" dr must be interpreted as potentially taking finite values when a discrete jump occurs (Harrison and Taksar, 1983; Harrison, 1985).

[18] It is worth noting that the above setup allows us to deal with more complex profit-sharing mechanisms. For example, suppose at V^* the regulator introduces a PS rule defined as a percentage cut of the firm's profits: we can model this rule adding a new stochastic differential equation for the profit cut. That is:

$$dV_t = \alpha V_t dt + \sigma V_t dW_t - dr_t,$$

and

$$dM_t = s dr_t$$

where M is defined by giving up $1/s$ unit of V for each unit of M. Threfore, above V^*, the PS rule can now be reformulated in terms of the new variable $Y_t = V_t/M_t$. When the existing combination of (V_t, M_t) places Y_t above s, the regulator intervenes immediately by cutting back on profits ($dr_t > 0$). The amount of profits cut is very small and is such as to push the firm's value along a line of slope $1/s$.

3 The optimal PS rule

In the previous section we have modelled the PS rule (5) for a given exogenous upper bound value V^*. We now need to define which value triggers the regulator's PS introduction as well as the regulator's adoption of the alternative strategy, the option to revoke the contract.

In this section we perfom our analysis under the simpler assumption of the firm's profit verifiability which - in this context - refers to the fact that the profit level can be proved in court: this implies that the regulator's threat of contract closure becomes binding for every firm's profit level higher than the optimal trigger.

We assume the regulator minimizes an intertemporal social welfare loss function which is an increasing function of the firm's profit: indeed, an increase in the monopolist's profits reduces the monetary value of consumer welfare. In this perspective, if the firm's profits becomes too high, that is, if the social welfare loss is too large, the regulator adopts one of the following alternative and equivalent strategies:

1. introducing a PS rule - defined as (5) - to divert profits from the firm to consumers;

2. revoking the contract and re-determiing provision, thus re-addressing the project's profitability.

Contract closure is then an "outside" option the regulator can always exercise. We model this option as a perpetual call option, with the project's value V as the underlying asset: thus, by considering a social welfare loss function, the regulator revokes the contract if V exceeds a critical threshold V^{**}.

In this section we prove that (5) is - under the assumption of profit verifiability - optimally determined by fixing $V^* = V^{**}$, which makes the regulator indifferent between revoking the contract and applying the PS rule. We obtain this result by firstly determining the value V^{**} that triggers revocation (*Proposition 1*), and then showing that V^{**} is indeed the regulator's optimal trigger to introduce the PS rule (5) (*Proposition 2*).

3.1 Social welfare and the option to revoke

The regulator's intertemporal loss function when the contract is revoked at time T is:[19]

$$L(V_T; V_0) + (I - V_T) \tag{6}$$

where $L(V_T; V_0)$ is the consumers' welfare reduction up to the revocation time T, V_0 is the value of the project at time zero, and $L_V(V_T; V_0) > 0$, $L(V_0; V_0) \geq 0$. The term $I - V_T$ is the regulator's (net) cost of revocation. Indeed, revocation is costly as the contract closure determines that the management of the project is back in the regulator's hands and this implies that the regulator should implement the new utility provision (i.e.: through direct management, privatization or contracting out to another firm). Specifically, this cost of revocation captures the regulator's cost in finding for a new franchisee, or - in the case of direct provision of the service - in training and hiring new personnel and/or adopting new technologies, or legal expenditure if the firm decides to sue the regulator, or, more generally, any cost belonging from regulatory capture from the firm.[20]

Since minimizing (6) is equivalent to maximizing $V_T - I - L(V_T; V_0)$, it is evident that rent extraction can be part of the regulator's objective in revoking the contract.[21] Exercising the option to revoke requires the payment of the sunk cost I plus the social cost $L(V_T; V_0)$. By the sunkness of I, it is never optimal to revoke when $V_T - I - L(V_T; V_0)$ is equal to zero, that is, for the regulator, it is better to wait until the value reaches a higher level.

Among the many possible ways of modelling the social welfare loss, we adopt a utilitarian criterion and approximate $L(V_T; V_0)$ as $\lambda(V_T - V_0)$, where $0 \leq \lambda \leq 1$ can be interpreted as the fiscal distortion in raising public funds if the service has to be run in-house.[22] Defining $F(V)$ as the value of the

[19] Our results will still hold if the regulator's preference also takes into account firm's value, but give it less weight than public funds.

[20] For a discussion about the different sources of regulatory capture from the firm, see Laffont and Tirole (1994, chapter 11).

[21] See Crew and Kleindorfer (1996, p. 218), for a discussion on rent extraction as included in the regulator's objective function.

[22] According to the utilitarian criterion we can approximate the welfare function at time T by the weighted and discounted average of the net surplus of consumers $K - (1 + \lambda)V_T$ and the value of the project V_T. Hence, the social welfare loss is simply given by:

$$L(V_T) \equiv K - \lambda V_0 - [K - \lambda V_T] = \lambda(V_T - V_0)$$

option at $t = 0$, we get:

$$F(V) = \max_T E_0 \left[(V_T - I - L(V_T))e^{-\rho T} \mid V_0 = V \right] \quad (7)$$
$$= \max_T E_0 \left[(1-\lambda)V_T - \hat{I})e^{-\rho T} \mid V_0 = V \right]$$

where $T(V^{**}) = \inf (t \geq 0 \mid V_t - V^{**} = 0^+)$ is the unknown future time when the option is exercised, V^{**} is the threshold value that triggers that action and $\hat{I} \equiv I - \lambda V_0$ is the exercise price. The optimization is subject to (3) and V_0[23].

Note that $F(V)$ is a perpetual call option. By using standard results in the (real) option valuation (Dixit and Pindyck, 1994), the solution of (7) is given by:

Proposition 1 *The value of the regulator's option to revoke at time $t \geq 0$ is given by:*

$$F(V) = \begin{cases} A(V^{**})V^{\beta} & \textit{for all } V < V^{**} \\ \\ (1-\lambda)V - \hat{I} & \textit{for all } V \geq V^{**} \end{cases} \quad (8)$$

where:

$$V^{**} = \frac{\beta}{\beta - 1}\frac{1}{1-\lambda}\hat{I}, \quad \textit{with} \quad \frac{\beta}{\beta-1} > 1^{24} \quad (9)$$

and:

$$A(V^{**}) = \frac{1-\lambda}{\beta}(V^{**})^{1-\beta} > 0 \quad (10)$$

Proof. see Appendix B ■

Hence, the regulator's optimal revocation rule can be expressed as: "**Revoke the contract as soon as the value of the project exceeds the adjusted break-even value** V^{**}".

Inspection of the opportunity cost λ in (9) reveals that:

---

where K is the expected value of the consumers' willingness to pay for the service (Laffont and Tirole, 1994).

[23] Moreover, for a consistent optimal revocation, we must also assume that $\hat{I} > 0$ and $V^{**} - \hat{I} > 0$.

[24] $\beta > 1$ is the positive root of the quadratic equation: $\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - \rho = 0$

- as $\lambda \to 0$, i.e. the regulator becomes socially "indifferent" between the direct management of the utility and the franchising contract to a firm, V^{**} drops to $\frac{\beta}{\beta-1} I$ and the probability of revocation increases.

- as $\lambda \to 1$, i.e. the opportunity cost of direct management by the regulator rises, $V^{**} \to \infty$ and the regulator never revokes.

To interpret (8), let's rewrite it in the following form:

$$
\begin{aligned}
F(V) &= \left[(1-\lambda)V^{**} - \hat{I}\right]\left(\frac{V}{V^{**}}\right)^{\beta} \\
&= \left[(1-\lambda)V^{**} - \hat{I}\right] E_0\left[e^{-\rho T}\right]
\end{aligned}
\tag{11}
$$

Maximizing (7) means maximizing the expected discounted value of the net benefit $(1-\lambda)V^{**} - \hat{I}$ when the utility is expropriated at time T, where $E_0\left[e^{-\rho T}\right] = \left(\frac{V}{V^{**}}\right)^{\beta} < 1$ is the expected discount factor. Then, maximizing (11) with respect to V^{**} gives the optimal revocation trigger as in (9).

3.2 Revocation vs Profit-Sharing

Since for $V_t > V^{**}$ it is optimal for the regulator to revoke the contract to minimize social welfare loss, in this section we demonstrate that V^{**} is indeed the optimal reflecting barrier for the regulator's introduction of the PS mechanism (5). That is, by setting $V^* = V^{**}$, the regulator is indifferent between applying the PS rule and revoking the contract.

First, we find the regulator's welfare loss function when the PS has been adopted. Denoting the expected value of future cumulative profit reduction due to (5) by $R(V_T; V^*)$, the regulator's loss function at T becomes:

$$
\lambda(V_T - V_0) + (I - V_T) + (1-\lambda)R(V_T; V^*) \tag{12}
$$

where V^* is a generic reflecting barrier. Next, the value of the regulator's option to revoke (7) at zero is now:

$$
F^r(V) = \max_T E_0^r\left[(1-\lambda)V_T - \hat{I} - (1-\lambda)R(V_T; V^*))e^{-\rho T} \mid V_0 = V\right] \tag{13}
$$

where the superscripts refer to the PS rule (5) and the optimization is subject to (4) and V_0. Solution of (13) shows that:

Proposition 2 *i) The regulator's optimal revocation trigger once the PS is adopted is equal to:*

$$V^{**} = \frac{\beta}{\beta - 1}\frac{1}{1-\lambda}\hat{I} \tag{14}$$

ii) If $V^* = V^{**}$, the PS rule (5) keeps the regulator indifferent to revocation, i.e.:

$$F^r(V_t) = 0 \qquad \textit{for } t \geq 0 \tag{15}$$

Proof. see Appendix C ■

Proposition 2 ascertains the optimality of the sharing rule (5) with respect to the regulator's alternative equivalent strategy, that is, the optimal regulator's contract closure: if the firm keeps profits below V^{**}, revocation is never optimal.

Proposition 2 provides further considerations. First, the optimal revocation trigger under PS is equal to the optimal revocation trigger without PS as in (9). This is just an application of the dynamic programming principle of optimality: if at $t = 0$ the regulator sets V^{**} as the optimal revocation trigger, this should be optimal for any $t > 0$, independently of any future policy after V^{**}.

Second, if the regulator sets $V^* = V^{**}$ as a reflecting barrier, the value of its option to revoke is always equal to zero. The intuition behind this relevant result is a straightforward implication of the barrier control r_t applied to the process V_t. Indeed, the true cost of exercising the option for the regulator is not just equal to the strike price $\hat{I}$, but also includes the future profit cuts $R(V_t; V^*)$ and the value of the forgone option $F^r(V_t)$. Thus, the net expected present value of optimal exercise at time $t \geq 0$ is:

$$E_t^r\left\{[(1-\lambda)V^* - \hat{I} - F^r(V^*)]e^{-\rho(t-T)}\right\} - (1-\lambda)R(V_t; V^*) = -F^r(V^*)\left(\frac{V_t}{V^*}\right)^\beta \tag{16}$$

where the last equality follows from $R(V_t; V^*) = [V^* - \frac{\hat{I}}{1-\lambda}]\left(\frac{V_t}{V^*}\right)^\beta$ (see Appendix C). Maximizing (16) with respect to V^* gives:

$$\beta\frac{F^r(V^*)}{V^*} - F^{r\prime}(V^*) = 0 \tag{17}$$

Since to avoid arbitrage at V^* the second term of (17) must be equal to zero, we get (15).[25] That is, from (16), the regulator is indifferent between introducing the PS rule and revoking the contract when the expected benefits from profit regulation exactly offset the expected social welfare loss due to the monopolist's excess profits.

Finally, although the PS rule (5) is simply proportional to the project's value, several novel implications follow:

- r_t is parametrized by the initial condition V^* which, in turn, depends on the revocation cost I and on the opportunity cost parameter λ. An increase in I and λ decreases r_t.

- r_t is non-decreasing and is given by the cumulative amount of profit cuts exerted on the sample path of V_t up to t. Thus, r_t relates to past realizations of V_t, which makes the PS time-dependent.

4 Efficiency of the PS

An important facet of the PS rule - analysed in the previous sections - is its dynamic sustainability when the firm's profit are observable, but nonverifiable. In this case, the regulator cannot force the firm to cut "excessive" profits as "no court or other third party will accept to arbitrate a claim based on the value taken by these variables" (Salanié, 1997, p.177). Then, given the assumption of profit's non-verifiability, is still the regulator's threat of contract closure in itself sufficient to induce the firm to comply with (5) as V_t intersects V^*? In this section we formally demonstrate that the proposed PS rule sustains a perfect equilibrium for the repeated continuous time regulatory relationship that starts at T^*: we do this by showing that any firm's deviation from (5) makes contract closure worthwhile for the regulator. In addition, since V_t is a Markov process, it is easy to ascertain that the equilibrium is also sub-game perfect.

[25] The function $F^r(V_t)$ is defined as the expected value of the regulator's net benefit when the utility is expropriated at time T. As the net benefit is a continuous function of the primitive process V_t, also F^r is a continuous function except perhaps when $V_t = V^*$ and the profit sharing rule r_t is applied. The behavior around $V_t = V^*$ is given by expanding $F^r(V_t)$ as:

$$F^r(V^*) = F^r(V^* - dr) = F^r(V^*) - F^{r\prime}(V^*)dr$$

which yields $F^{r\prime}(V^*) = 0$. This condition holds at any reflecting barrier without any optimization being involved (Dixit, 1993).

The regulatory game we consider here lasts a possibly infinite[26] number of periods, and ends once the regulator exercises the option to revoke the franchise contract. Each period is divided in four stages: at the first stage, the regulated monopolist is delegated to manage the supply of a public utility and nature chooses a parameter determining the profit of the firm. At the second stage, after having observing the firm's profit, the regulator decides whether or not to ask for a PS: if the regulator perceives that the monopolist is making "excessively" high profits, he sets a profit ceiling, say V^*, according to (9), above which the PS rule (5) applies; the regulator accompanies its announcement with a threat to revoke the contract if the firm does not comply.[27]. At the third stage, the monopolist decides whether or not to comply with the regulator's prescription (i.e.: that is, whether or not to start with stream of payment $r_t \geq 0$)[28]. At the fourth stage, the regulator, conditional on the price sets by the monopolist, decides whether or not to revoke the contract. If the regulator does revoke, the monopolist suffers the loss V_t and the regulator obtains $V_t - I$ (i.e.: the net gain from revocation). If the regulator does not revoke, the game goes ahead to the next period and it is repeated.

However, without a binding commitment by the regulator, any finite number of firm profit reductions will be inefficient. In fact, the regulator's problem is that for any $t \geq T^*$ he has an incentive to carry out his threat, even if the monopolist reduces its profits. Since this means that the monopolist will not ward off the threat by reducing its profits, the monopolist will not reduce them. Thus, the unique sub-game perfect equilibrium is inefficient: revocation is carried out regardless of the monopolist's positive net present value. To avoid this inefficiency the firm must continuously "control" its profits; that is, for $t \geq T^*$, the monopolist should consider V^* as the ceiling not to be crossed, and reduce its expected profits just enough to keep $V_t < V^*$ to prevent contract closure.[29] To summarize:

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[26] Owing to uncertainty, neither firm nor regulator can perfectly predict V_t each time. As V_t follows a random walk, for each time interval dt there is a constant probability of moving up or down. Therefore, the game ends in finite (stochastic) time with probability one, but everything proceeds as if the horizon were infinite.

[27] By the Markov Property of (22), in our model it is not important when the regulator announces V^* as long as it is between zero and T^*.

[28] In our infinite-lived project without investment, the firm's dominant strategy is not to make the payment $r_t \geq 0$, that is not to comply.

[29] There are many efficient sub-game perfect equilibria where the threat of revocation induces an infinite flow of payments by the firm to prevent contract closure (see Shavell

Proposition 3 *The following strategy represents a sub-game perfect equilibrium:*

i) As long as $V_t < V^*$ nothing is done. As soon as V_t crosses V^* from below, the monopolist reduces its profits by (5) and the regulator does not revoke.

ii) Profit regulation ends in finite (stochastic) time with probability one.

Proof. see Appendix D ∎

As both the players expect an infinite repetition of their relationship, their choices in each period will depend on the previous moves. The players' strategy for each period $t \geq T^*$ can be described as follows: the monopolist observes V_t and chooses to impose a profit reduction r_t or not; the regulator "Does not Revoke" if the firm has followed the rule r_t to keep $V_t < V^*$ for all $t' < t$[30]. On the contrary, the regulator "Revokes" if the firm has deviated from r_t at any $t' < t$.

Our stochastic-continuous time framework calls for an instantaneous reply by the regulator when the monopolist departs from the PS rule (5), that is, the regulator adopts the most severe punishment: revocation.[31] The regulator believes that this mechanism, from the initial date and state (T^*, V^*), will be retained for the whole (stochastic) planning horizon. Since the project we model here is infinitely-lived, the present value of forgone profits will indeed always ensure participation by the firm, and the expectation of future profit regulations keeps the regulator from carrying out his threat.[32]

Finally, the second part of the *Proposition 3* says that with probability one the profit regulation ends in the (stochastic) finite interval. Intuitively, although the firm prefers to cut profits rather than terminate the contract (i.e. the loss from closure is greater than the expected profit cuts), it always

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and Spier, 1996, Proposition 2).

[30] In our continuous time setting we assume, without any loss of generality, that when the regulator is indifferent between revoking the contract and not revoking, he does not exercise the option.

[31] In continuous time repeated games there is no notion of *last time before t*, so induction cannot be applied. For examples on how to represent continuous time as a sequence of discrete-time grids that becomes infinitely negligible, we refer the reader to Simon and Stinchcombe (1989) and Bergin and MacLeod (1993).

[32] Considering a long-term, but finite, franchise contract, the firm's opportunistic behaviour in the last period of the contract should be taken into account: in this case, the firm's incentive into comply with the regulator's PS prescription are different from that we have modelled here.

prefers to stop payment if the threat of revocation is not carried out. The firm "regulates" profits until $V_t \geq V^*$according to r_t, but when V_t reaches, for the first time after T^*, the trigger V^* again, it ceases regulation. That is, at $T'^* = \inf(t \geq T^* \mid V_t - V^* = 0^-)$, if the firm sets $r_{T'} = 0$, the regulator will face a jump from zero to $F(V^*)$ but revocation is not carried out (See Figure 1). The game then starts afresh.

5 PCR and dynamic price adjustment

Let's conclude showing how the above PS rule can be implemented in a PCR setting. To do this we introduce a reduced form for the firm's profit function (1) that depends only on the price of the service and a demand shock, i.e. $\mathbf{z}_t = (p_t, \theta_t)$. That is, we assume that:

1. The market demand at time t is a constant-elasticity function of the price p_t:
$$D(p_t) = \theta_t p_t^{-\varepsilon} \tag{18}$$
with $\varepsilon \in (0,1)$.

2. The random parameter θ_t follows a trendless, geometric Brownian motion, with instantaneous volatility $\sigma > 0$, i.e.:
$$d\theta_t = \sigma\theta_t dW_t, \qquad \text{with} \;\; \theta_0 = \theta \tag{19}$$
where dW_t is the standard increment of a Wiener process[33].

3. No operating costs are present but there is a per period fixed cost c[34]. Then, the monopolist's project gives a profit flow at each time t equal to[35]:
$$\pi(p_t, \theta_t) = v(p_t, \theta_t) - c \equiv p_t^{1-\varepsilon}\theta_t - c. \tag{20}$$

[33] By the Markov Property of (19), the quality of all subsequent results does not change if we assume a trend for demand to increase.

[34] The fixed costs we consider here are, as standard in the literature, flow fixed costs of production: that is, we assume that the firm begins the first period endowed with a technology whose operation entails a flow cost c per unit of time.

[35] We avoid, for simplicity, considering operating options such as reducing output or even shutting down that incresaes the value of the firm (MacDonald and Siegel, 1986; Dixit and Pindyck,1994, chs. 6 and 7).

4. The monopolist is subject to price regulation. The price p is allowed to increase by the difference between expected inflation (the Retail Price Index, RPI) and an exogenously given expected increase in productivity over time (x):

$$dp_t = (RPI - x)p_t dt, \qquad \text{with } \; p_0 = p \tag{21}$$

These assumptions enable us to reduce the model to one dimension. Expanding $d\pi(p_t, \theta_t)$ and applying the Itô's lemma it is easy to show that $v(p_t, \theta_t)$ is driven by the following geometric Brownian motion:

$$dv_t = \alpha v_t dt + \sigma v_t dW_t \quad \text{with } v_0 = v, \tag{22}$$

with:

$$\alpha \equiv (1 - \varepsilon)(RPI - x)$$

The drift parameter of the process v_t is a linear combination of the corresponding parameters of the primitive process θ_t and of the price-cap rule (21). Finally, since the monopolist is risk-neutral, using the simplified expression for the profit function (22), the market value of the project becomes:

$$V = \frac{v}{\rho - \alpha} - \frac{c}{\rho} \tag{23}$$

As far as the price-cap revision is concerned, in the event of the firm's profits going beyond a "pre-determined" level, the PS rule requires the x factor to be automatically adjusted upward, making the price-cap adjustment rate $RPI - x$ more stringent (Sappington, 2002). According to this practice we can rewrite (4) as:

$$dV_t = (1 - \varepsilon)(RPI - x')V_t dt + \sigma V_t dW_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \tag{24}$$

where $x' = x - \frac{d \inf\limits_{0 \le v \le t} (V^*/V_v)/dt}{(1-\varepsilon) \inf\limits_{0 \le v \le t} (V^*/V_v)} > x$ is the new price decrease factor which stops the process V_t from going any higher than V^*. How the x' factor works seems intuitively appealing. As the numerical value for V^* is known, by (23) the optimal revocation trigger (9) can be written as $p_t^{1-\varepsilon}\theta_t = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\frac{\rho-\alpha}{\rho}(c + \rho\hat{I})$, from which the boundary value for θ^* is determined by:

$$\theta^*(p_t) = \frac{\beta}{\beta - 1}\frac{1}{1 - \lambda}\frac{\rho - \alpha}{\rho}\frac{c + \rho\hat{I}}{p_t^{1-\varepsilon}} \tag{25}$$

For any given value of the price p_t, random fluctuations of θ_t move the point (θ_t, p_t) horizontally left or right. If the point goes to the right of the boundary, then a price reduction is made immediately shifting the point down to the boundary. If θ_t stays to the left of the boundary, no new price reduction is applied. Thus, price reduction proceeds gradually to maintain (25) the equality. To illustrate, suppose $RPI - x = 0$ so that $p_t = p_0$ for all t; by inverting (25) we obtain the optimal boundary function $p(\theta_t)$ which determines the optimal price regulation as a function of the sole state variable θ_t:

$$p_t = p_0 \left(\frac{\theta^*}{\theta_t}\right)^{1/1-\varepsilon} \qquad \text{with} \quad \frac{dp_t}{d\theta_t} < 0 \tag{26}$$

Futhermore, higher costs shift the boundary (25) to the right, $\theta'^* > \theta^*$ and determine firm's smaller price reduction to comply with the PS rule[36]. The boundary function for this case is shown in Figure 2 below.

Figure 2 - about here -

6 Final Remarks

In this paper we employ a real options approach to investigate the properties of a second-best optimal profit-sharing (PS) mechanism imposed by a welfare-maximising regulator. We consider a dynamic setting where a regulated monopolist is delegated to manage a long-term franchise contract to supply a public utility. If the monopolist's profit becomes "excessively" high, that is, if the social welfare loss is too large, we have assumed that the regulator always has the possibility to adopt one of the following alternative and equivalent strategies:

- introducing a PS mechanism to divert profits from the firm to consumers;
- revoking the contract and re-determing provision, thus re-addressing the project's profitability.

Specifically, we have modelled the regulator's option to revoke as a perpetual call option which is a function of the firm's profits, social welfare and the regulator's cost of revocation.

[36] As matter of fact, cost padding by the franchisee is another strategy that might be used to avoid the appearance of excess profit. It would be possible to model the franchisee as reporting costs and the regulator as employing auditors to determine the accuracy of cost reports, but this is not our topic here.

Under the assumption of verifiability of profits, we have endogenously determined the profit threshold that triggers revocation (*Proposition 1*) and proved that this threshold keeps the regulator indifferent between revoking the contract and applying the PS rule (*Proposition 2*). We formally show that in the unique equilibrium of the game the monopolist will always comply with the regulator's PS rule and the contract closure will never occur; the regulator will impose the PS rule whenever the profit is higher than the identified trigger level.

Under the more realistic assumption of profit's non-verifiability, we have then investigated the dynamic sustainaibility of the PS clause. We formally proved that - for all the profit levels higher that the profit threshold which makes the regulator indifferent between contract closure and imposing the PS - it is sequentially optimal for the regulator to revoke the contract: that is, we showed that any firm's deviation from the PS rule makes revocation worthwhile for the regulator. Hence, the perfect equilibrium of the game with profits non-verifiability is also such that the monopolist complies with the PS rule chosen by the regulator in each period, as long as the revocation has not been carried out (*Proposition 3*). The price adjustment which follows results thus endogenously and dynamically determined as the monopolist's best response to the regulator's choice.

We have also showed that as the regulator's contract closure can be very costly - i.e. it could imply costs belonging from any form of capture of the regulator by the firm - a considerable regulatory lag can occur before a PS rule is introduced: the higher the revocation cost, the lower the profit shared and the less frequent the regulator's PS prescription. This conclusion suggests a theoretical reason why the PS mechanism in its dynamic application would tend to be unsuccessful in the real world: specifically, as long as the regulator's threat to revoke the contract becomes not credible, the regulated firm is no longer rewarded for comply with the adopted PS rule.

Finally, introducing a reduced form of the firm profit function, we have provided the x factor adjustment in a price-cap regulation setting: this application illustrates how the parameters which characterize the profit function affect the PS effectiveness.

To close, let us briefly suggest a possible extension of our analysis which recalls the main symplifying assumption we adopted in the paper. The economic literature on PS regulation generally holds that profit-sharing rules reduce the firm's incentive to invest (Lyon, 1996; Crew and Kleindorfer, 1996; Sappington, 2002; Moretto et al., 2003). In contrast with this literature, our analysis - which address specifically the unilateral regulator's bargaining position in the regulatory contract - has been carried out under the assumption of no firm investment. Thus, a natural extention of our model could include the firm's choice of investment - both irreversible and reversible - to assess the effects of contract closure by the regulator on such firm's strategic decision.

Appendix

A. The regulation mechanism

We define the regulation which follows the introduction of the PS rule as the reduction dV_t needed to keep V_t at V^*. This is represented by a one-sided non-decreasing adapted control process (as in Harrison, 1985) on the state variable V which is right-continuous and non-negative. Then, the control policy consists of a process $Z = \{Z_t, t \geq 0\}$ and a regulated process $V^r = \{V_t^r, t \geq 0\}$ such that:

$$V_t^r \equiv V_t Z_t, \quad \text{for} \quad V_t^r \in (0, V^*], \tag{27}$$

where:

- $i)$ V_t is a geometric Brownian motion, with stochastic differential as in (3);
- $ii)$ Z_t is a decreasing and continuous process with respect to V_t ;
- $iii)$ $Z_0 = 1$ if $V_0 \leq V^*$, and $Z_0 = V^*/V_0$ if $V_0 > V^*$ so that $V_0^r = V^*$;
- $iv)$ Z_t decreases only when $V_t^r = V^*$.

Applying Ito's lemma to (27), we get:

$$d\,V_t^r = \alpha\,V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}, \qquad V_0^r \in (0, V^*]$$

where $V_t^r \frac{dZ_t}{Z_t} \equiv V_t dZ_t = -dr_t$ is the infinitesimal level of value forgone by the firm. In terms of the regulated process V_t^r, we can write:

$$r_t \equiv r(V_t) = V_t - V_t^r \equiv (1 - Z_t)V_t, \tag{28}$$

Although the process Z_t may have a jump at time $t = 0$, it is continuous and keeps V_t below the barrier V^* exercising the minimum amount of control: in fact, control is exercised only when V_t crosses V^* from below with a probability one in the absence of regulation. Therefore, in the case of $V_0 < V^*$, we get $V_t^r \equiv V_t$, with the initial condition $V_0^r \equiv V_0 = V$, and $Z_t = 1$.

At $T^* \equiv T(V^*) = \inf(t \geq 0 \mid V_t - V^* = 0^+)$ control starts so as to maintain $V_t^r = V^*$.

The firm's values are adjusted downward by the amount $r_t = V_t - V_t^r \geq 0$ every time V^* is hit. The same conditions $(i)-(iv)$ uniquely determine Z_t with the representation form (Harrison,1985; Proposition 3, p. 19-20):[37]

$$Z_t \equiv \begin{cases} \min(1, V^*/V_0) & \text{for } t = 0 \\ \inf\limits_{0 \leq v \leq t} (V^*/V_v) & \text{for } t \geq 0 \end{cases} \tag{29}$$

B. Proof of Proposition 1

The function $F(V_t)$ is defined as the expected value at time t of the regulator's net benefit when the utility is expropriated at time T. As the net benefit is a continuous function of the primitive process V_t , also F is a continuous function of V_t. Then, by a short arbitrage argument (Cox and Ross, 1976; Harrison and Kreps, 1979), applying Ito's lemma to F, the value of the regulator's option to revoke becomes the solution of the following differential equation (Dixit and Pindyck, 1994, p. 147-152):

$$\frac{1}{2}\sigma^2 V_t^2 F''(V_t) + \alpha V_t F'(V_t) - \rho F(V_t) = 0, \qquad \text{for } V_t \in (0, V^{**}], \tag{30}$$

where $F(V_t)$ must satisfy the following boundary conditions:

$$\lim_{V_t \to 0} F(V_t) = 0 \tag{31}$$

$$F(V^{**}) = (1-\lambda)V^{**} - \hat{I} \tag{32}$$

$$F'(V^{**}) = 1 - \lambda \tag{33}$$

If the value of the utility goes to zero, so does the option. Conditions (32) and (33) imply respectively that, at the trigger level V^{**}, the value of the option is equal to its liabilities where $\hat{I}$ indicates the sunk cost of revocation (matching value condition) and suboptimal exercise of the option is ruled out (smooth pasting condition). By the linearity of (30) and using (31), the general solution is of the form:

$$F(V_t) = AV_t^{\beta}, \tag{34}$$

[37] This is an application of a well-known result of Levy (1948), for which the process:

$$\ln V_t^r \equiv \ln V_t + \ln Z_t \equiv \ln V_t - \inf_{0 \leq v \leq t} (\ln V_v - \ln V^*)$$

has the same distribution as the "reflected Brownian process" $\mid \ln V_t - \ln V^* \mid$.

where A is a constant to be determined and $\beta > 1$ is the positive root of the quadratic equation:

$$\Phi(\beta) = \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - \rho = 0 \tag{35}$$

As (34) represents the option value of optimally revoking the contract, the constant A must be positive and the solution is valid over the range of V_t for which it is optimal for the regulator to keep the option alive $(0, V^{**}]$. By substituting (34) for (32) and (33) we get:

$$V^{**} = \frac{\beta}{\beta - 1}\frac{1}{1 - \lambda}\hat{I} \ , \tag{36}$$

and:

$$A(V^{**}) \equiv \left[(1 - \lambda)V^{**} - \hat{I}\right](V^{**})^{-\beta} \equiv \frac{1 - \lambda}{\beta}(V^{**})^{1-\beta} > 0. \tag{37}$$

This concludes the proof.

C. Proof of Proposition 2

We prove that when the regulator uses (28), its option to revoke is always equal to zero.

Cost of regulation

Let's denote by $R(V_t^r; V^*)$ the expected value of future cumulative profit cuts. At $t = 0$ this is given by:

$$\begin{aligned} R(V_0; V^*) \ &\equiv \ E_0^r\left\{\int_0^\infty e^{-\rho t} dr(V_t) \mid V_0^r \equiv V_0 \in (0, V^*]\right\} \\ &= \ -E_0^r\left\{\int_0^\infty e^{-\rho t} V_t dZ_t] \mid V_0^r \equiv V_0 \in (0, V^*]\right\} \end{aligned} \tag{38}$$

where V^* is the (generic) upper reflecting barrier defined in (27). Since V_t^r is a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81), we know that the foregoing conditional expectation is in fact a function of the starting state alone.[38] Keeping the

[38] For $V_0 = V > V^*$ optimal control would require Z to have a jump at zero so as to ensure $V_0^r = V^*$. In this case the integral on the right of (38) is defined to include the control cost r_0 incurred at $t = 0$, that is (see Harrison 1985, p. 102-103):

$$\int_0^\infty e^{-\rho t} dr_t \equiv r_0 + \int_{(0,\infty)} e^{-\rho t} dr_t$$

where $r_0 = V - V_0^r$.

dependence of R on V_t^r active and assuming that it is twice continuously differentiable, by Itô's lemma we get:

$$\begin{aligned} dR &= R' dV_t^r + \frac{1}{2} R''(dV_t^r)^2 \\ &= R'(Z_t dV_t + V_t dZ_t) + \frac{1}{2} R'' Z_t^2 (dV_t)^2 \\ &= R'(\alpha V_t^r dt + \sigma V_t^r dW_t + V_t^r \frac{dZ_t}{Z_t}) + \frac{1}{2} R'' Z_t^2 \sigma^2 dt \\ &= \frac{1}{2} R'' \sigma^2 V_t^{r2} dt + R' \alpha V_t^r dt + R' \sigma V_t^r dW_t + R' V_t^r \frac{dZ_t}{Z_t} \end{aligned} \tag{39}$$

where it has been taken into account that for a finite-variation process like Z_t, $(dZ_t)^2 = 0$. As $dZ_t = 0$ except when $V_t^r = V^*$ we are able to rewrite (39) as:

$$dR(V_t^r; V^*) = [\frac{1}{2}\sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha V_t^r R'(V_t^r; V^*)]dt \tag{40}$$

$$+\sigma V_t^r R'(V_t^r; V^*)\, dW_t - R'(V^*; V^*) dr(V_t)$$

This is a stochastic differential equation in R. Integrating by part the process Re^{-rt} we get (Harrison, 1985, p. 73):

$$e^{-\rho t} R(V_t^r; V^*) = R(V_0; V^*) + \tag{41}$$

$$+ \int_0^t e^{-\rho s} \left[\frac{1}{2}\sigma^2 V_s^{r2} R''(V_s^r; V^*) + \alpha V_s^r R'(V_s^r; V^*) - \rho R(V_s^r; V^*) \right] ds$$

$$+\sigma \int_0^t e^{-\rho s} V_s^r R'(V_s^r; V^*)\, dW_s - R'(V^*; V^*) \int_0^t e^{-\rho s} dr(V_s)$$

Taking the expectation of (41) and letting $t \to \infty$, if the following conditions apply:

(a) $\lim_{l \to 0} \Pr[T(l) < T(V^*) \mid V_0 \in (0, V^*]] = 0$ for $l \leq V_t^r < V^* < \infty$, where $T(l) = \inf(t \geq 0 \mid V_t^r = l)$ and $T(V^*) = \inf(t \geq 0 \mid V_t^r = V^*)$;

(b) $R(V_t^r;V^*))$ is bounded within $(0,V^*]$;

(c) $e^{-\rho t}V_t^r R'(V_t^r;V^*)$ is bounded within $(0,V^*]$;

(d) $R'(V^*;V^*)=1$;

(e) $\frac{1}{2}\sigma^2 V_t^{r2}R''(V_t^r;V^*)+\alpha V_t^r R'(V_t^r;V^*)-\rho R(V_t^r;V^*)=0$,

we obtain $R(V^r;V^*)$ as indicated in (38). Condition (a) says that the probability of the regulated process V_t^r reaching zero before reaching another point within the set $(0,V^*]$ is zero. As V_t^r is a geometric type of process this condition is, in general, always satisfied (Karlin and Taylor, 1981, p. 228-230). Furthermore, if condition (a) holds and $R(V^r;V^*)$ is bounded, then conditions (b) and (c) also hold. According to the linearity of (e) and using (d), the general solution has the form:

$$R(V_0;V^*)=B(V^*)(V_0)^\beta, \tag{42}$$

with:

$$B(V^*)=\frac{1}{\beta}(V^*)^{1-\beta}>0 \tag{43}$$

As for $V_0\le V^*$, $Z_0=1$ and $V_0^r\equiv V_0=V$, then $R(V_0;V^*)=R(V;V^*)$. On the other hand, if $V_0>V^*$, we get $Z_0=V^*/V_0$, so that $V_0^r=V^*$ and $R(V_0^r;V^*)=R(V^*;V^*)$.

The value of revocation under profit control

Indicating by $F^r(V)$ the regulator's value of the option under profits control, this can be expressed, at time zero, by:

$$F^r(V)=\max_T E_0^r\left[((1-\lambda)V_T-\hat{I}-(1-\lambda)B(V^*){V_T}^\beta)e^{-\rho T}\mid V_0=V\right] \tag{44}$$

As in (34) this takes the form:

$$F^r(V)=AV^\beta$$

If the regulator decides for revocation, the optimal threshold, say V^{**}, must satisfy the two familiar conditions:

$$A(V^{**})^\beta=(1-\lambda)V^{**}-\hat{I}-(1-\lambda)B(V^*)(V^{**})^\beta \tag{45}$$

$$\beta A(V^{**})^{\beta-1} = (1-\lambda) - (1-\lambda)\beta B(V^{*})(V^{**})^{\beta-1} \tag{46}$$

These two equations can be solved for the trigger V^{**} and for the constant A. Simple algebra shows that V^{**} is independent of B and then of the barrier V^{*}. The solution is:

$$V^{**} = \frac{\beta}{\beta-1}\frac{1}{1-\lambda}\hat{I}$$

and the constant A is equal to:

$$\begin{aligned} A &= -(1-\lambda)B(V^{*}) + \frac{(1-\lambda)}{\beta}(V^{**})^{1-\beta} \\ &= (1-\lambda)\left[\frac{1}{\beta}(V^{*})^{1-\beta} - \frac{1}{\beta}(V^{**})^{1-\beta}\right] \end{aligned}$$

Therefore setting $V^{**} = V^{*}$ the constant $A = 0$ and the option value is identically equal to zero.

Finally, as r_t depends only on the primitive exogenous process V_t, the "regulated" process $V_t - r_t$ is also a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81). Thus, any option value beginning at a point t at which revocation has not taken place has the same solution. This concludes the proof of Proposition 2.

D. Proof of Proposition 3

We prove that the regulatory scheme proposed in Proposition 2 is also optimal when the regulator cannot force the firm to adopt it. We proceed in the following way. First, since by Proposition 2 the sharing rule r_t makes the regulator's option to revoke the contract always equal to zero, it is also a good candidate for supporting a long-run equilibrium of the threat-game. Next, we prove that this is indeed the case by applying a sort of one-stage-deviation principle and showing that any deviation from r_t makes revocation worthwhile (the non-decreasing property of r_t is crucial to this result). Finally, the Markov property of the "regulated" process $V_t - r_r$ makes the equilibrium sub-game perfect.

Revocation strategy and perfect equilibrium

It is well known that infinitely repeated games may be equivalent to repeated games that terminate in finite time. At each period there is a probability that the game continues one more period. The key is that the conditional probability of continuing must be positive (Fudenberg and Tirole, 1991, p. 148). This is indeed our case, neither player can perfectly predict V_t at any date and the sharing rule (28) with form (29) is viewed by both agents as a stationary strategy for evaluating all future profit reductions.[39] In the strategy space of the regulator it appears as:

$$\phi(V_t, r_t) = \begin{cases} \text{Do not revoke at } t = T^* \text{ if the firm} \\ \text{plays the rule } r_t = (1 - Z_t)V_t \text{ for } t' < t \\ \\ \text{Revoke if the firm deviates from} \\ r_t = (1 - Z_t)V_t \quad \text{at any } t' < t \end{cases}$$

where $\phi(V_t, r_t)$ is the strategy at t with history (V_t, Z_t). The regulator's "threat" strategy is chosen if the firm deviates by adjusting V_t less than r_t or by abandoning $r_t = (1 - Z_t)V_t$ as a rule to evaluate future adjustments. The regulator must believe that the regulation, from the initial date and state (T^*, V^*), will be kept in use for the whole (stochastic) planning horizon. If the firm deviates, the regulator believes that the firm intends to switch to a different rule in the future and knows for sure that the regulator will revoke immediately thereafter. The regulator does not revoke at t if $r_{t'} \geq V_{t'} - V_{t'}^r$ for

[39] Integrating the differential form (3), the geometric Brownian motion can be expressed as:

$$V_{t+dt} = V_t e^{dY_t}$$

where $dY_t = \mu dt + \sigma dW_t$ and $\mu = \alpha - \frac{1}{2}\sigma^2$. The differential dY_t is derived as the continuous limit of a discrete-time random walk, where in each small time interval of length Δt the variable y either moves up or down by Δh with probabilities (Cox and Miller, 1965, p. 205-206):

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\sqrt{\Delta t}}{\sigma}\right), \quad \Pr(\Delta Y = -\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\sqrt{\Delta t}}{\sigma}\right)$$

or defining $\Delta h = \sigma\sqrt{\Delta t}$:

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\Delta h}{\sigma^2}\right), \quad \Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\Delta h}{\sigma^2}\right)$$

That is, for small Δt, Δh is of order of magnitude $O(\sqrt{\Delta t})$ and both probabilities become $\frac{1}{2} + O(\sqrt{\Delta t})$, i.e. not very different from $\frac{1}{2}$. Furthermore, considering again the discrete-time approximation of the process Y_t, starting at $V^* e^{+\Delta h}$, the conditional probability of reaching V^* is given by (Cox and Miller, 1965, ch.2):

$$\Pr(Y_t = 0 \mid Y_t = 0 + \Delta h) = \begin{cases} 1 & \text{if } \mu \leq 0 \\ e^{-2\mu\Delta h/\sigma^2} & \text{if } \mu > 0 \end{cases}$$

which converges to one as Δh tends to zero.

all $t' \leq t$, because profit cuts are expected to continue with the same rule and $F^r(V) = 0$ for all $t \geq T^*$. If $r_{t'} < V_{t'} - V_{t'}^r$ for some $t' < t$ the regulator expects a different rule and carries out the threat, switching from $F^r(V_t) = 0$ to $F(V_t) \geq V^* - I$. The game is over. To prove this, we first need to prove the following Lemma:

Lemma 4 *For each $t' > T^*$ we get:*

$$R(V_{t'}; V^*) = (\rho - \alpha) E_{t'}^r \int_{t'}^{\infty} e^{-\rho(s-t')} r_s ds \tag{47}$$

Proof. Let's consider R as in (38). For each $t' > T^*$, integration by parts gives:

$$\int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = \tag{48}$$

$$e^{-\rho(t-t')} V_t Z_t - V_{t'} Z_{t'} + \rho \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds - \int_{t'}^{t} e^{-\rho(s-t')} Z_s dV_s$$

Taking the expectations of both sides and using the zero-expectation property of the Brownian motion (Harrison, 1985, p. 62-63), we have:

$$E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = E_{t'}^r [V_t Z_t e^{-\rho(t-t')}] - V_{t'} Z_{t'} + (\rho - \alpha) E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds \tag{49}$$

By the Strong Markov property of V_t^r[40], it follows that $E_{t'}^r [V_t Z_t e^{-\rho(t-t')}] = E_{t'}^r [V_t Z_t] E_{t'}^r [e^{-\rho(t-t')}] = V^* E_{t'}^r [e^{-\rho(t-t')}] \rightarrow 0$ almost as surely as $t \rightarrow \infty$, so that:

$$E_{t'}^r \int_{t'}^{\infty} e^{-\rho(s-t')} V_s dZ_s = -V_{t'} Z_{t'} + (\rho - \alpha) E_{t'}^r \int_{t'}^{\infty} e^{-\rho(s-t')} (V_s - r_s) ds$$

Since $-V_{t'} Z_{t'} + (\rho - \alpha) E_{t'}^r \int_{t'}^{\infty} e^{-\rho(s-t')} V_s ds = 0$, substituting (38) and rearranging we get:

$$R(V_{t'}; V^*) = (\rho - \alpha) E_{t'}^r \int_{t'}^{\infty} e^{-\rho(s-t')} r_s ds$$

■

We now prove that r_t is sub-game perfect by showing that the firm cannot gain by deviating from r_t in an arbitrarily short interval and conforming to r_t thereafter. In

[40]The Strong Markov Property of regulated Brownian motion processes stresses the fact that the stochastic first passage time t and the stochastic process V_t^r are independent (Harrison, 1985, Proposition 7, p. 80-81).

particular, let us assume (t', t) an interval in which r_s is kept flat at $r_{t'}$ so that $V_s^r \leq V^*$, and t is the first time in which $dZ_t > 0$.

Considering the decomposition (49) we can write (47) as:

$$\begin{aligned} R(V_{t'}; V^*) &= (\rho - \alpha)\left\{E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'}^r \left\{\int_{t}^{\infty} e^{-\rho(s-t')} r_s ds\right\}\right\} \\ &= (\rho - \alpha)\left\{E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'}^r \left\{e^{-\rho(t-t')} \int_{t'}^{\infty} e^{-\rho(s-t')} r_s^* ds\right\}\right\} \end{aligned}$$

where we have defined $V_s^{r*} = V_{t+s}^r$ and $r_s^* = r_{t+s} - r_t$ for $t' \leq t$. Applying, again, the Strong Markov Property of V_t^r we get:

$$\begin{aligned} R(V_{t'}; V^*) &= E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'}^r \left\{e^{-\rho(t-t')} E_{t'}^r \int_{t'}^{\infty} e^{-\rho(s-t')} r_s^* ds\right\} \\ &= (\rho - \alpha) E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + E_{t'}^r \left\{e^{-\rho(t-t')} R(V_{t'}; V^*)\right\} \\ &= (\rho - \alpha) E_{t'}^r \int_{t'}^{t} e^{-\rho(s-t')} r_s ds + R(V_{t'}; V^*) E_{t'}^r \left\{e^{-\rho(t-t')}\right\} \end{aligned}$$

where the second equality follows from the assumption that $r_s = r_{t'} \equiv V_{t'} - V_{t'}^r$ for all $s \in (t', t)$. Finally, noting that $e^{-\rho(t-t')} \simeq 1 - \rho(t - t')$ and $\int_{t'}^{t} e^{-\rho(s-t')} ds \simeq (t - t')$ we can simplify the above expression as:

$$R(V_{t'}; V^*) \simeq \frac{(\rho - \alpha)}{\rho} r_{t'} \equiv \frac{(\rho - \alpha)}{\rho} (V_{t'} - V_{t'}^r) \tag{50}$$

From (50), any application of controls $r_{t'} < V_{t'} - V_{t'}^r$ leads to a reduction of (47) for all $t \geq t'$ and, then, by Proposition 2, to $F^r(V_t; V^*) > 0$ which triggers revocation by the regulator.

Furthermore, the firm does not adjust by more than r_t since doing so would not increase the probability of delaying revocation. It does not pay less, since $r_t < V_t - V_t^r$ induces closure making it worse off, i.e. $0 < V_t$.

Finally, as $V_t^r \equiv V_t - r_r$ is a Markov process in levels, it is clear from (47) that any sub-game that begins at a point at which revocation has not taken place is equivalent to the whole game. The strategy ϕ is efficient for any sub-game starting at an intermediate date and state (t, V_t). This concludes the first part of the Proposition.

Non-decreasing path of r_t within $[T^*, T'^*)$.

So far we have implicitly assumed that, once started at T^*, the profit-sharing goes on forever. Earlier interruptions are not feasible as long as the threat of revocation is credible. Credibility relies on the fact that the agency's option to revoke if the firm deviates from r_t is always worth exercising at $V_t > V^*$. However, in a Brownian path there is a positive probability of the primitive process V_t crossing V^* again starting at an interior point of the range (V^*, ∞). In this case, the firm may be willing to stop cutting profits. That is, the firm "regulates" profits until $V_t \geq V^*$according to r_t, but when V_t reaches, for the first time after T^*, a predetermined level, say $V' \leq V^*$, it ceases regulation. The regulator will then face a jump from zero to $F(V') \leq F(V^*)$ making the threat of revocation no longer credible.

To see how this happens let's assume that the firm stops adjusting profits at time T' with $T^* < T^{'} < \infty$, and $T' = \inf(t \geq T^* \mid V_t \geq V' \text{ and } V' \leq V^*)$, i.e. T' is the first time that the primitive process V_t reaches $V' \leq V^*$ with profit regulation under way. Then the value of the revocation option starting at any $t \in [T^*, \infty)$ with $t < T^{'}$ can be expressed as:

$$\tilde{F}(V_t, V_t^r; V') = P(V'; V_t) E_t^r[F(V_{T'}) e^{-r(t-T')}] + \tag{51}$$

$$(1 - P(V'; V_t)) E_t^r[F^r(V_{T'}) e^{-r(t-T)}]$$

where $P(V'; V_t)$ is the probability of the primitive process V_t reaching $V' \leq V^*$ starting at an interior point of the range (V^*, ∞), which is equal to (Cox and Miller, 1965, p. 232-234):

$$\Pr(T' < \infty \mid V_t) \equiv P(V'; V_t) = \left(\frac{V_t}{V'}\right)^{-2\mu/\sigma^2}$$

with $\mu = (\alpha - \frac{1}{2}\sigma^2)$[41]. As the starting point is now any $t \in [T^*, \infty)$, we can immediately see in (51) the dependence on both V_t^r and V_t.

Since the option value under profit regulation is zero, if V' is never reached we get $F^r(V_{T'}) = 0$. On the contrary, if V' is reached and the contract is revoked, it is simply $F(V_{T'}) = F(V')$, and:

$$\tilde{F}(V_t; V') = P(V'; V_t) E_t[F(V') e^{-r(T'-t)}]$$

According to the Strong Markov Property of V_t equation (51) becomes:

[41] This probability is $P(V'; V_t) = 1$ for $\mu \leq 0$.

$$\tilde{F}(V_t; V') = P(V'; V_t) F(V') \left(\frac{V_t}{V'} \right)^{\gamma} \tag{52}$$

where $\gamma < 0$ is the negative root of (35). Since at t the primitive process V_t is greater than V' and $P(V'; V_t) \left(\frac{V_t}{V'}\right)^{\gamma} = \left(\frac{V_t}{V'}\right)^{\gamma - 2\mu/\sigma^2} \leq 1$, we obtain $\tilde{F}(V_t; V') \leq F(V')$ for all $t \in [T^*, T')$, which implies that the following inequality holds:

$$\tilde{F}(V_t; V') = F(V^*) \left(\frac{V'}{V^*} \right)^{\beta} \left(\frac{V_t}{V'} \right)^{\gamma - 2\mu/\sigma^2} \leq F(V^*) \tag{53}$$

Since $\tilde{F}(V_t; V') \leq F(V^*)$ for all $t \in [T^*, T')$, the regulator's optimal strategy is to revoke immediately at T^*. To prevent revocation the profit adjustment must continue until time $T'^* \equiv T'(V^*) = \inf(t \geq T^* \mid V_t - V^* = 0^-)$ when the trigger value $V' = V^*$ is reached for the first time after T^*. The game ends and can then be started afresh. This concludes the second part of the Proposition.

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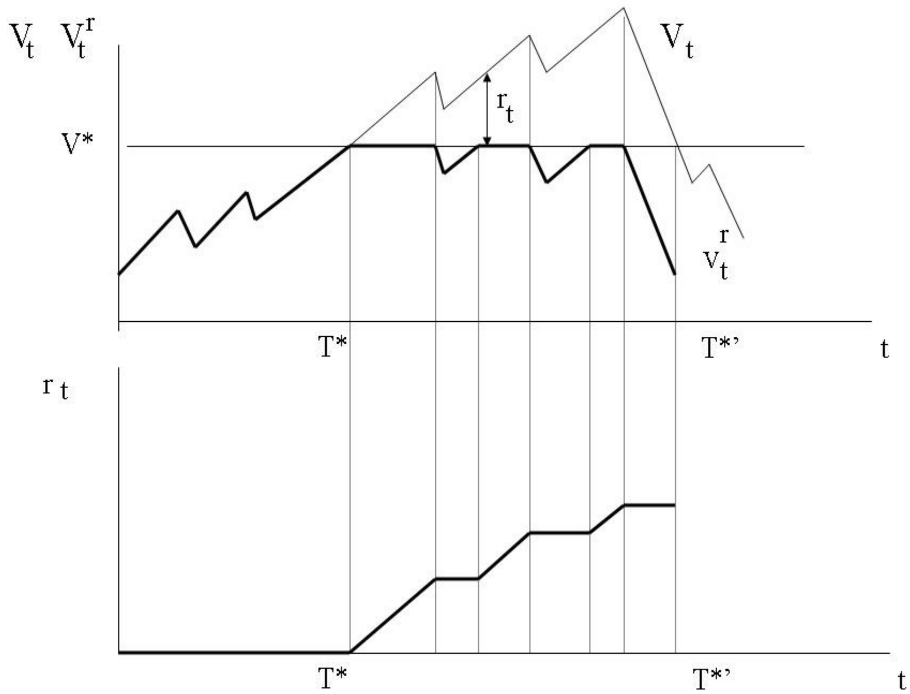

Figure 1: The firm's value- (i.e.: profit-) sharing dynamic

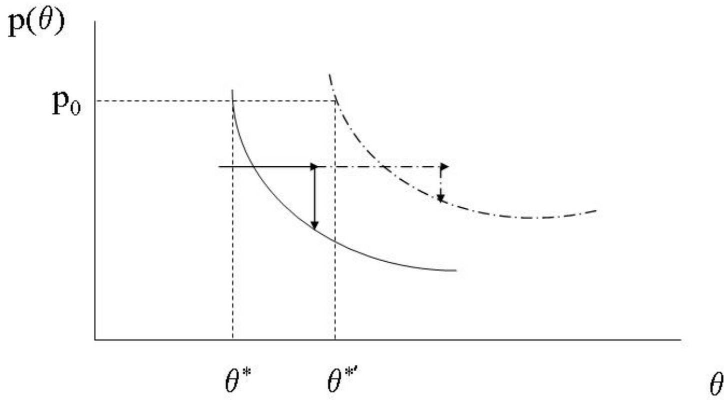

Figure 2: Price regulation